***Linear Differential Equations with Constant Coefficients***

**Linear Differential Equations:** A differential equation of the form,

… … .. (1)

where, , , … and are functions of *x* or, constants, is called a linear differential equation of nth order.

If , , … are all constants (not functions of  *x*) and is function of *x* or constant, then the equation is called a linear differential equation with constant coefficients.

If the right-hand term (non-homogeneous term) is identically zero, then the equation reduces to,

… … .. (2)

and it is called a linear homogeneous differential equation.

The general solution of equation (2) will be,

**a).**if the roots *i.e,* are real and distinct.

**b).**if the roots *i.e,* are real and equal.

**c).**if the roots are complexand distinct.

**d).**if the roots are complexand repeated.

**Problem-01:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,























The general solution is,



where, , , are arbitrary constants.

**Problem-02:** Solve



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,























The general solution is,



where, , , are arbitrary constants.

**Problem-03:** Solve



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,













The general solution is,



where, , are arbitrary constants.

**Problem-04:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,



















The general solution is,



where, , , are arbitrary constants.

**Problem-05:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,



















The general solution is,













where, , are arbitrary constants.

**Problem-06:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,



















The general solution is,













where, , are arbitrary constants.

**Problem-07:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,





















The general solution is,









where, , , , are arbitrary constants.

**Problem-08:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,

























The general solution is,



where, , , are arbitrary constants.

**Problem-09:** Solve.



**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,

















The general solution is,









where, , , , are arbitrary constants.

**Problem-10:** Solve.

**Solution:** Given that,

… … … (1)

Let, be the trial solution.

The auxiliary equation of (1) is,

















The general solution is,



where, , , ,are arbitrary constants.

**Exercise:**

**1.** Solve

**2.**Solve

**3.**Solve

**4.**Solve

**5.**Solve

**6.**Solve

**7.**Solve

**8.**Solve

***Linear Differential Equations with Constant***

***Coefficients but Right Side non-zero.***

Consider a differential equation of the form,

… … .. (1)

where, , , … are all constants (not functions of  *x*) and is function of *x* or constant but .

The general solution of equation (1) is,



whereis known as the complementary function (C.F) and is called the particular integral (P.I).

**Working Rules for Finding Particular Integral:**

1. .
2. if .
3. if .
4. 

**Exceptional case:**

1. if but .

Again, if  , then

.

1. if  but .

Again if but then

.

**NOTE:**

1. and .

**2.** 

**3.** 

**4.** 

**5.** 

**6.** 

**Problem-01:** Solve.

**Solution:** Given that,

… … … (1)

Let, be the trial solution of the corresponding homogeneous equation,

… … … (2)

The auxiliary equation of (2) is,

















The complementary function of (1) is,



The particular integral of (1) is,













Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-02:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

(2)

The auxiliary equation of (2) is,

















The complementary function of (1) is,



The particular integral of (1) is,















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-03:** Solve.

**Solution:** Given that,

(1)

Let, be the trial solution of the corresponding homogeneous equation,

(2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-04:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-05:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,













The complementary function of (1) is,



The particular integral of (1) is,















Therefore, the general solution of equation (1) is,





where, , ,,are arbitrary constants.

**Exercise: Try Yourself**

1.  **Ans:**
2.  **Ans:**
3.  **Ans:**

**Problem-06:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,















The complementary function of (1) is,



The particular integral of (1) is,







Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-07:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,















The complementary function of (1) is,



The particular integral of (1) is,









Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-08:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,





















The complementary function of (1) is,



The particular integral of (1) is,











Therefore, the general solution of equation (1) is,





where, , , are arbitrary constants.

**Problem-09:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,









Therefore, the general solution of equation (1) is,





where, , , are arbitrary constants.

**Problem-10:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,





















The complementary function of (1) is,



The particular integral of (1) is,











Therefore, the general solution of equation (1) is,





where, , , are arbitrary constants.

**Exercise: Try Yourself**

1. **Ans:**
2. **Ans:**
3. **Ans:**
4. **Ans:**

**Problem-11:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,













The complementary function of (1) is,



The particular integral of (1) is,









Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-12:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,



















The complementary function of (1) is,



The particular integral of (1) is,

















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-13:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,

















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-14:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,















The complementary function of (1) is,



The particular integral of (1) is,

















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-15:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,













The complementary function of (1) is,



The particular integral of (1) is,



















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-16:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,













The complementary function of (1) is,



The particular integral of (1) is,



















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Exercise: Try Yourself**

1.  **Ans:**
2.  **Ans:**
3.  **Ans:** 
4.  **Ans:** 
5.  **Ans:** 
6.  **Ans:** 
7.  **Ans:** 
8.  **Ans:**



**Problem-17:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,















The complementary function of (1) is,



The particular integral of (1) is,





















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-18:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,



















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-19:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,



























The complementary function of (1) is,



The particular integral of (1) is,























Therefore, the general solution of equation (1) is,





where, , ,are arbitrary constants.

**Problem-20:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,



















The complementary function of (1) is,



The particular integral of (1) is,

















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-21:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,















The complementary function of (1) is,



The particular integral of (1) is,





















Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-22:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,





























Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-23:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,











The complementary function of (1) is,



The particular integral of (1) is,

































Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-24:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,















The complementary function of (1) is,



The particular integral of (1) is,

































Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-25:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,









The complementary function of (1) is,



The particular integral of (1) is,































Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Problem-26:** Solve.

**Solution:** Given that,

 (1)

Let, be the trial solution of the corresponding homogeneous equation,

 (2)

The auxiliary equation of (2) is,









The complementary function of (1) is,



The particular integral of (1) is,



































Therefore, the general solution of equation (1) is,





where, , are arbitrary constants.

**Exercise: Try Yourself**

1. **Ans:**

***Linear Differential Equations with variables***

***Coefficients***

An equation of the form



where,  are constants and is function of *x* or constant, is called the linear differential equation with variables coefficients.

**NOTE:** If we put , then the equation (1) is transformed into an equation with constant coefficients changing the independent variable from *x* to *t* as,



Now







Again, 















Similarly, 

… … … … … … … … … … … … …



From (1) we get,



The equation (2) is a linear differential equation with constant coefficients.

**Problem-01:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,







Let, be the trial solution of the equation (2)

Then the auxiliary equation of (2) is,

















The general solution of (1) is,







where, , are arbitrary constants.

**Problem-02:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,









Therefore the general solution is,





where, , are arbitrary constants.

**Problem-03:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,











Therefore the general solution is,





where, , are arbitrary constants.

**Problem-04:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,











The complementary function of (1) is,





The particular integral of (1) is,











Therefore the general solution is,





where, , are arbitrary constants.

**Problem-05:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,























Therefore the general solution is,





where, , are arbitrary constants.

**Problem-06:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,









Now let, 





which is linear equation

Therefore, 





Integrating,







Again, 







which is also a linear equation

Therefore, 





Integrating,









Therefore the general solution is,





where, , are arbitrary constants.

**Problem-07:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,









Let , 





which is linear equation

Therefore, 





Integrating,















Again, 





which is also a linear equation

Therefore, 





Integrating,





















Therefore the general solution is,





where, , are arbitrary constants.

**Exercise: Try Yourself:**

**01:** Solve 

**02:** Solve 

**03:** Solve 

**04:** Solve 